

$\pi^+\pi^+$ and $\pi^+\pi^-$ colliding in noncommutative space

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Abstract

By studying the scattering process of scalar particle pion on the noncommutative scalar quantum electrodynamics, the non-commutative amendment of differential scattering cross-section is found, which is dependent of polar-angle and the results are significantly different from that in the commutative scalar quantum electrodynamics, particularly when $\cos\theta \sim \pm 1$. The non-commutativity of space is expected to be explored at around $\Lambda_{NC} \sim \text{TeV}$.

PACS number(s): 11.10.Nx, 25.80.Dj, 13.85.-t.

Keywords: Noncommutative field theory, pi-meson, High-energy elastic scattering.

1 Introduction

The emergence of the noncommutative geometry with a neutral way in string theory/(M-Theory) in a definite limit not only makes us effectively analyze the duality, BPS state and D-brane dynamics, but also causes a revolution in the whole physical theory because a fresh idea – noncommutative spacetime was put forward^[1–10]. This means that in the Planck-scale the space-time coordinates are no more commutative and they will satisfy an uncertain-relation, the space-time point loses its original sense and the geometry which describes the original physical phenomena is not consistent with the new physics in this space-time area. Therefore, it is necessary to have a new space-time geometry – noncommutative space-time geometry to describe the gravitation^[11–23]. Thus, there have been a lot of delightful achievements in the noncommutative field theory, the topological phase and the correction of noncommutative energy levels in recent years^[24–28]. In addition, Refs.[29,30] have studied the Wigner function in noncommutative space.

In this paper we focus on the scattering process of scalar particle pion on the noncommutative scalar quantum electrodynamics(NCSQED). In section 2, we mainly review the Weyl-Moyal product in noncommutative space. In section 3 we give the Feynman rules for NCSQED and show that there are some differences between the quadric kinematic terms in NCSQED and in noncommutative electrodynamics(NCED). In section 4 and 5 we discuss the elastic scattering

processes of $\pi^+\pi^+$ and $\pi^+\pi^-$ respectively. The annihilation of the scalar particles is studied in section 6, and there are discussions on three and four-point photon functions ($\theta^{0i} \neq 0$). Conclusion and further discussion are given in the last section.

2 The Weyl-Moyal product in noncommutative space

In this part we mainly review the Weyl-Moyal product in noncommutative space. As is known that the study of noncommutative field theory arose from the pioneering work by Snyder^[1]. In recent years, with the development of string theory, it has been paid more attention and great progresses have been made. For example, noncommutativity of space is an important characteristic of D-brane dynamics at low energy limit^[2,3]. What's more, interests are increasing in possible physically observable consequences related to non-commutativity of space, which is expected to justify noncommutative field theory and noncommutative quantum mechanics with the references of seminal works^[4–10]. In general, noncommutativity relation of space-time can be described as:

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = \frac{ic_{\mu\nu}}{\Lambda_{\text{NC}}^2} \quad (1)$$

where $\theta_{\mu\nu}$ is an anti-commutation parameter and of dimension $[L]^2$, Λ_{NC} is the scale, in this scale the space-time is no longer commutative. The action for field theory in noncommutative space is then obtained by using the Weyl-Moyal correspondence^[11]. Thus, in order to find the noncommutative action the usual product of field should be replaced by the star-product,

$$(f * g)(x) = \exp \left[\frac{i}{2} \theta_{\mu\nu} \partial_{x_\mu} \partial_{x_\nu} \right] f(x)g(y)|_{x=y}, \quad (2)$$

in which f and g are two arbitrary infinitely differentiable functions on R^{3+1} . This product has an important character,

$$\int d^4x (f \star g)(x) = \int d^4x (g \star f)(x) = \int d^4x (f \cdot g)(x). \quad (3)$$

This means that if the free Lagrangian only include quadric field variable, we can construct the same Fock space as in commutative space to perform the perturbative evaluation^[11]. In the case of $\theta^{0i} = 0$, it has been shown that noncommutative ψ^4 theory up to two loops^[11,12] and noncommutative QED up to the one loop^[13,14] are renormalizable. While ($\theta^{0i} \neq 0$), it has been shown that the Noncommutative field theory is not unitary. So, the theory is not appealing^[15].

Noncommutative action can also be obtained according to Seiberg-Witten(SW) map^[16]. Using this SW-map, Calmet and others first constructed a model with noncommutative gauge invariance which was close to the usual standard model and is known as the minimal NCSM(mNCSM). They also listed several Feynman rules^[17]. Although many phenomenological studies^[18–22] have

been made to unravel several interesting features of this mNCSM, Raimar Wulkenhaar has proved that “*Noncommutative scalar QED cannot be renormalized by means of Seiberg-Witten expansion.*”^[23]. He has provided some ideas for how the Seiberg-Witten expansion can be used as a computational technique to treat one-loop divergences of the θ -unexpanded noncommutative QED.

3 Feynman rules for NCSQED

With the revision of Weyl-Moyal product we provide Feynman rules for NCSQED in this section. Assuming Raimar Wulkenhaar’s word^[10] also makes sense for noncommutative scalar quantum electrodynamics, we study the full electromagnetic scattering of noncommutative scalar quantum electrodynamics. The full lagrangian in noncommutative field theory can be obtained by replacing the usual product with the star product. So, the lagrangian for the scalar noncommutative can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu} \star F^{\mu\nu} + (D_\mu\phi)^* \star (D_\mu\phi), -m^2\phi^* \star \phi, \quad (4)$$

which is invariant under the following transformations

$$\phi(x, \theta) \rightarrow \phi'(x, \theta) = U \star \phi(x, \theta), \quad (5)$$

$$A_\mu(x, \theta) \rightarrow A'_\mu(x, \theta) = U \star A_\mu(x, \theta) \star U^{-1} + \frac{i}{e}U \star \partial_\mu U^{-1} \quad (6)$$

where $U = (e^{i\alpha})_\star$ and

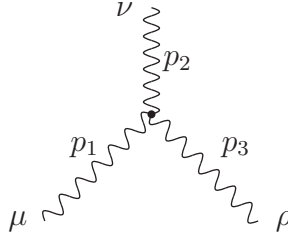
$$D_\mu\phi = \partial_\mu\phi - ieA_\mu \star \phi, \quad (7)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie(A_\mu \star A_\nu - A_\nu \star A_\mu). \quad (8)$$

Obviously, besides the photon’s 2-point function(which is identical to the ordinary case because its forms remain unchanged (see Eq.(3))), the photon’s 3- and 4-point functions are now transparent. After transforming these new interactions into momentum space, the vertices pick up additional phase factors which are dependent upon the momenta flowing through the vertices. These kinematic phases play an important role in the colliding process of the noncommutative field theory. Now that All Feynman rules are in Ref.(5), here we only mention Feynman rules which will be used in this paper. The photon’s vertices are same as of the NCQED, and Feynman rules for the photon’s 3-point vertex are shown explicitly in Fig.1, where $p \wedge k = p \cdot \theta \cdot k$ is used as notation.

The interacting Lagrangian of scalar particles and photons can be written as

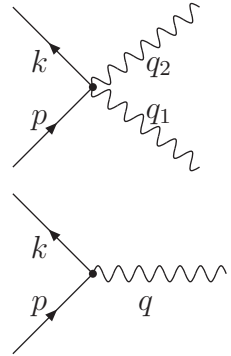
$$\mathcal{L}_{S-B} = ieA^\mu(\phi^* \star \partial_\mu\phi - \phi \star \partial_\mu\phi^*) - e^2 A^\nu \star A_\nu \star \phi^* \star \phi. \quad (9)$$



$$= 2e \sin(p_1 \wedge p_2) [(p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu}]$$

Figure 1: The Feynman rules for 3-point vertex

Obviously, different from the 3- and 4-point functions of photons, the very modification of noncommutativity of space-time is the additional kinematic phase factor. Thus, according to Eq.(9) we can easily obtain Feynman rules for these interactions in Fig.2. (It should be noted that there are two ways to calculate Feynman rules for the second term of Eq.(8), so an average should be made.)



$$= 2ie^2 g_{\mu\nu} \cos(p \wedge k) \exp\left[\frac{i}{2} q_1 \wedge (p + k)\right]$$

$$= -ie(k + p)_\mu \exp\left(\frac{i}{2} p \wedge k\right)$$

Figure 2: The Feynman rules for interactions of scalar particles to photon

4 $\pi^+\pi^+$ elastic scattering in NCSQED

As the Möller and Bhabha scattering in the non-commutative electrodynamics ^[6], the reaction $\pi^+\pi^+ \rightarrow \pi^+\pi^+$ and $\pi^+\pi^- \rightarrow \pi^+\pi^-$ is a simple model which can be used to test the non-commutativity of space in the NCSED. However, there are some differences from the NCQED due to the quadric kinematic terms in the Lagrangian. This can be realized explicitly from the interaction Lagrangian (8), as well as from the Feynman rules in Fig.2. These differences are expected to guide a new approach to test the noncommutativity of space during the colliding process of high-energy particles.

Let us first consider the elastic $\pi^+\pi^+$ colliding, whose Feynman diagrams are displayed in Fig.3. In this case, kinematic phase corresponding to NC modifications appears in each vertex.

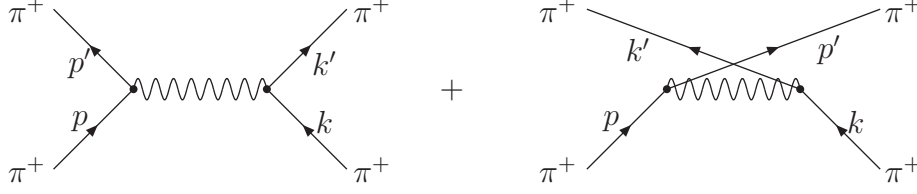


Figure 3: The scattering of $\pi^+\pi^+$

Following Feynman rules in Fig.2 and momentum labels in Fig.3, we can get the invariant matrix element of the scattering process

$$\begin{aligned}
i\mathcal{M}_{++} = & -ie^2 \exp \left[\frac{i}{2}(p \wedge p' + k \wedge k') \right] (p + p')^\alpha (k + k')^\beta \frac{g_{\alpha\beta}}{q^2} \\
& - ie^2 \exp \left[\frac{i}{2}(p \wedge k' + k \wedge p') \right] (p + k')^\alpha (k + p')^\beta \frac{g_{\alpha\beta}}{q^2}.
\end{aligned} \tag{10}$$

Obviously, t - and u -channel interference terms hold a factor $\cos \delta$ where δ is the phase difference of t and u -channel,

$$\delta_{++} = \frac{1}{2}(p \wedge p' + k \wedge k') - \frac{1}{2}(p \wedge k' + k \wedge p'). \tag{11}$$

It should be noted that all terms involving time-space noncommutativity have been dropped out of the expression of δ_{++} because this theory is not unitary and hence, as a field theory, it is not appealing^[10]. All other effecting quantities are shown in the following formulas (in which center-of-mass frame and Mandelstam variables are used.),

$$p \wedge p' = \frac{s}{4}(\theta_{12} \sin \theta \cos \phi - \theta_{31} \sin \theta \sin \phi), \tag{12}$$

$$p \wedge k' = \frac{s}{4}(-\theta_{12} \sin \theta \cos \phi + \theta_{31} \sin \theta \sin \phi), \tag{13}$$

$$k \wedge p' = \frac{s}{4}(-\theta_{12} \sin \theta \cos \phi + \theta_{31} \sin \theta \sin \phi), \tag{14}$$

$$k \wedge k' = \frac{s}{4}(\theta_{12} \sin \theta \cos \phi - \theta_{31} \sin \theta \sin \phi). \tag{15}$$

Using these formulas we can simplify the expression of δ

$$\delta_{++} = -\sqrt{ut}(\theta_{12} \cos \phi - \theta_{31} \sin \phi). \tag{16}$$

In addition, as we take the limit $\Lambda_{NC} \rightarrow \infty$, $\cos \delta = 1$ the ordinary result is recovered. Using the Mandelstam variables we can obtain the squared invariant matrix element after a simple calculation

$$|\mathcal{M}_{++}|^2 = \left(\frac{4\pi\alpha}{t} \right)^2 \left[(s-u)^2 + (s-t)^2 + 2 \cos \delta (s-u)(s-t) \right]. \tag{17}$$

While, in the high-energy limit ($m \rightarrow 0$) the Mandelstam variables have the following simple forms

$$s = (2E)^2 = E_{\text{CM}}^2 \quad (18)$$

$$u \approx -2E^2(1 + \cos \theta) \quad (19)$$

$$t \approx -2E^2(1 - \cos \theta). \quad (20)$$

Here the note E is the energy of the incoming particle. What's more, the differential cross section can be deduced

$$\left(\frac{d\sigma_{++}}{d\Omega}\right)_{\text{CM}} = \frac{|\mathcal{M}_{++}|^2}{64\pi^2 s^2}. \quad (21)$$

In numerical analysis, we set the machine energy for $\Lambda_{\text{NC}} = \sqrt{s} = 1.2 \text{ TeV}$. We only consider one non-vanishing value of θ_{ij} and set its magnitude for $1/\Lambda_{\text{NC}}^2$. Further, for simplicity we just consider the case $\theta_{31} = \theta_{12} = 1/\Lambda_{\text{NC}}^2$. In this way, when $\phi = \pi/4$, the results will be obviously same as in the commutative case. In Fig.(4), we plot the angular distribution of the ratio between the noncommutative modification and the differential cross section, $(d\sigma - d\sigma_{\text{NC}})/d\Omega$. We find that the ratio vanish at $\theta = 0, \pi$, i.e. there is no noncommutative modification in this case.

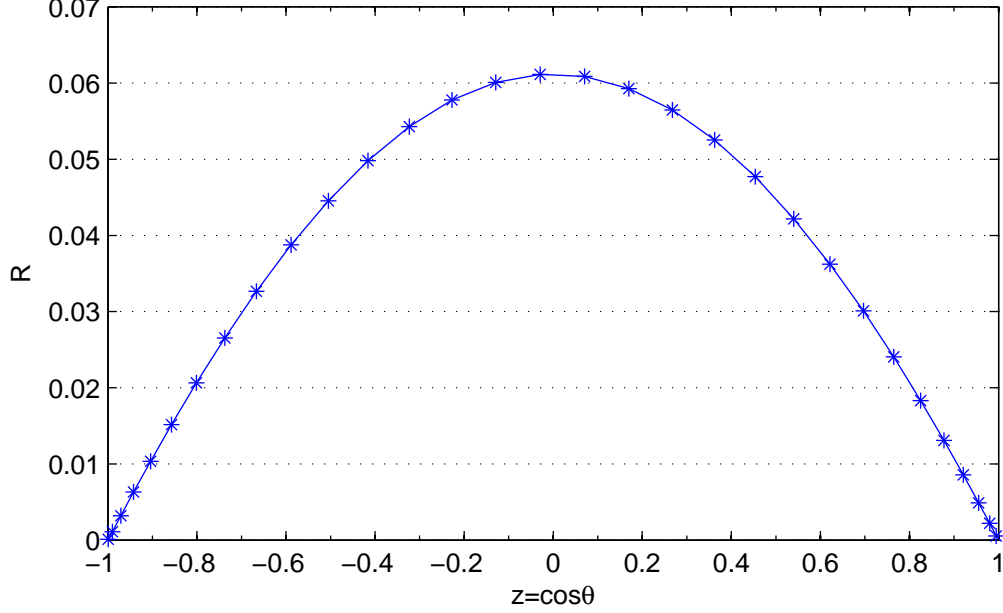


Figure 4: The ratio $(d\sigma - d\sigma_{\text{NC}})/d\Omega$ at $\phi = \pi/2$ (real line) and π (*-dotted) for scattering $\pi^+\pi^+ \rightarrow \pi^+\pi^+$. The reason of superposition for the two different cases is that the noncommutative energy scale is so large that the dependency of phase difference to ϕ become unimportant.

5 $\pi^+\pi^-$ elastic scattering in NCSQED

Now that we have completed our discussion of the process $\pi^+\pi^+ \rightarrow \pi^+\pi^+$, let us consider a different but similar process $\pi^+\pi^- \rightarrow \pi^+\pi^-$. The lowest order Feynman diagram is pictured in Fig.5. Similar to the process of $\pi^+\pi^+ \rightarrow \pi^+\pi^+$, following the momentum labeling given in

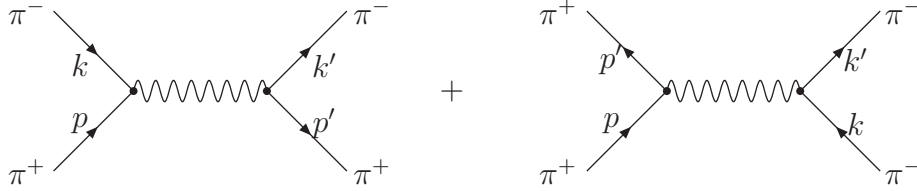


Figure 5: The Feynman diagram of elastic scattering of $\pi^+\pi^- \rightarrow \pi^+\pi^-$.

Fig.5, we can write down the matrix element as

$$\begin{aligned}
 i\mathcal{M}_{+-} = & -ie^2 \exp \left[-\frac{i}{2}(p \wedge k + p' \wedge k') \right] (p - k)^\alpha (k' - p')^\beta \frac{g_{\alpha\beta}}{q^2} \\
 & + ie^2 \exp \left[\frac{i}{2}(p \wedge p' + k \wedge k') \right] (p + p')^\alpha (k + k')^\beta \frac{g_{\alpha\beta}}{q^2}.
 \end{aligned} \tag{22}$$

We can see that the s - and t -channel interference terms now pick up a kinematic phase difference given by

$$\delta_{+-} = \frac{1}{2}(p \wedge k + p' \wedge k') + \frac{1}{2}(p \wedge p' + k \wedge k'). \tag{23}$$

Neglecting the effect of noncommutativity of space-time, i.e. $\theta_{0i} = 0$, and using the formulas of (12-15,18-20), explicitly we can obtain this phase difference

$$\delta_{+-} = \sqrt{ut}(\theta_{12} \cos \phi - \theta_{31} \sin \phi) = -\delta_{++}. \tag{24}$$

After a straightforward calculation we can obtain the squared invariant matrix element

$$|\mathcal{M}_{+-}|^2 = \left(\frac{4\pi\alpha}{s} \right)^2 \left[(s+u)^2 + (s+t)^2 + 2 \cos \delta_{+-} (s+u)(s+t) \right]. \tag{25}$$

The differential cross section of this process having the same form as of (21) can be expressed by

$$\left(\frac{d\sigma_{+-}}{d\Omega} \right)_{\text{CM}} = \frac{|\mathcal{M}_{+-}|^2}{64\pi^2 s^2}. \tag{26}$$

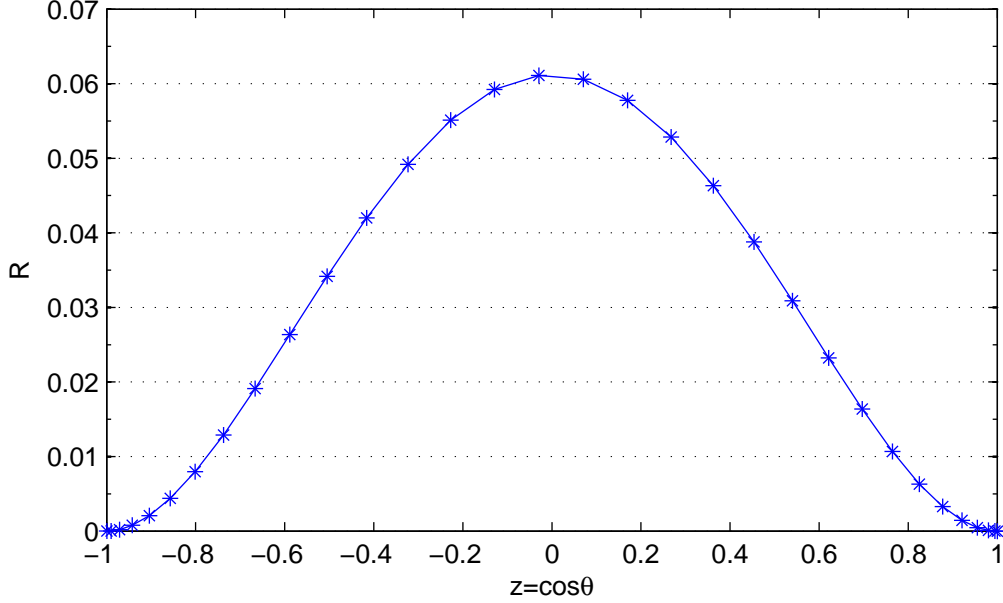


Figure 6: The ratio $(d\sigma - d\sigma_{\text{NC}})/d\Omega$ at $\phi = \pi/2$ (real line) and π (*-dotted) for scattering $\pi^+\pi^- \rightarrow \pi^+\pi^-$. As the same case and reason there is a superposition for the two different cases. However, at $\theta = 0, \pi$ the change rates become slow compared to the previous reaction.

6 Annihilation of $\pi^+\pi^-$ in NCSED

In this section we will study the annihilation process of $\pi^+\pi^-$ in NCSED. It is well known that some Feynman diagrams have contributed a lot to this process. For examples, the 3-point and 4-point photon functions contradict with 2-point photon function. But it is a pity that all these diagrams contain the photon-loop, we have to ignore those contributions made. The Feynman diagrams of this annihilation up to tree level are draft out in Fig.7. Due to the presence of the non-Abelian-like coupling, we must be careful in calculation of the cross section to ensure that the Ward identities are satisfied so that the unphysical polarization states are not produced. Meanwhile, we have to note that this process is sensitive to space-time noncommutativity. So, the cross section will not be invariant in all reference frames because of the violation of Lorentz invariance which is a character of the noncommutative field theory^[7].

Using the Feynman rules given in section 1, we can now read the invariant matrix element

$$\begin{aligned}
i\mathcal{M}_{\text{ap}}^{\mu\nu} = & 2e^2 \exp \left[\frac{i}{2} p \wedge k \right] (p+k)_\rho \frac{1}{q^2} \sin(k' \wedge p') [(k' - p')^\rho g^{\mu\nu} + (p' - q)^\mu g^{\nu\rho} + (q - k')^\nu g^{\rho\mu}] \\
& - ie^2 \exp \left[-\frac{i}{2} (p \wedge p' + k \wedge k') \right] \frac{(p+q)^\mu (k+q)^\nu}{q^2}
\end{aligned}$$

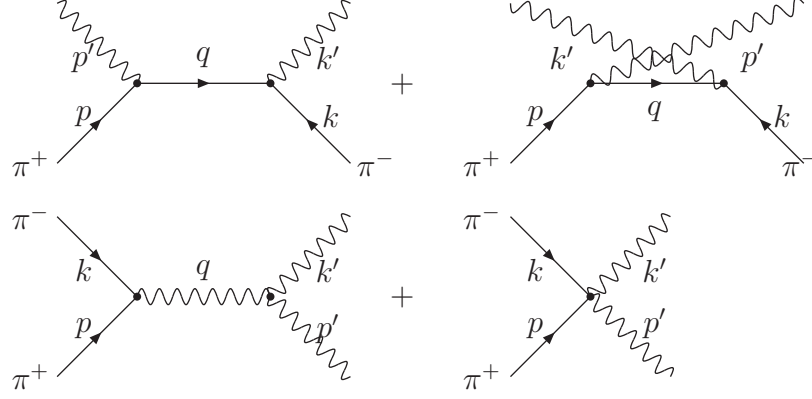


Figure 7: The Feynman diagrams of annihilation of $\pi^+\pi^-$

$$\begin{aligned}
& -ie^2 \exp \left[-\frac{i}{2}(p \wedge k' + k \wedge p') \right] \frac{(p+q)^\mu (k+q)^\nu}{q^2} \\
& + 2ie^2 g_{\mu\nu} \cos(p \wedge k) \exp \left[\frac{i}{2} k' \wedge (p+k) \right].
\end{aligned} \tag{27}$$

If $\theta_{0i} = 0$, i.e. only the effect of noncommutativity of space-space is considered, the first line of (27) will vanish. Accordingly, the phase difference of the u - and t -channel will vanish. That is to say, there are no additional factors to the interference terms between u - and t -channels, and the last term will be a constant as the original case. Therefore, if there is some influence arising from the modifications of noncommutative field theory, the noncommutativity of space-time must be put into consideration. As the Noncommutative Electrodynamics, the Noncommutative Scalar Electrodynamics is also sensitive to the noncommutativity of space-time.

As this is an unappealing theory in the case $\theta_{0i} \neq 0$, we don't intend to deduce the scattering cross section, although it is possible to establish a consistent noncommutative theory in which the scattering matrix is unitary via a new definition of the Time-order in perturbation theory^[24].

7 Conclusion remarks

In this paper we have analyzed various $2 \rightarrow 2$ high energy scattering processes in the noncommutative scalar electrodynamics. As the modifications of electrodynamics, both result in a non-Abelian-like nature with 3- and 4-point photon self-couplings, as well as momentum dependent phase factors appearing at each possible vertex in noncommutative scalar QED. However, there are some differences from the noncommutative QED due to the quadric kinematic terms in the lagrangian, which can be realized explicitly from the interaction lagrangian (9) or can be understood from the Feynman rules in Fig.2. We have known that the interference of the distinct channels plays a significant role. If there is only one channel of reaction, the effect of

noncommutativity will be hidden for tree level diagram, although there are new Feynman rules of 3- and 4-point photon self-couplings for the case $\theta_{0i} \neq 0$ as we discussed in the last section. In the numerical analysis, we have parametrized the noncommutative relationship in terms of NC-energy-scale Λ_{NC} and an antisymmetric unit matrix $c_{\mu\nu}$, i.e. $\theta_{ij} = c_{ij}/\Lambda_{NC}^2$. We hope that these results can be used, as a new approach, to test the non-commutativity of space.

8 Acknowledgements

This work was supported by the National Natural Science Foundation of China (10965006 and 11175053), an open topic of the State Key Laboratory for Superlattices and Microstructures (CHJG200902), the scientific research project in Shaanxi Province (2009K01-54) and Natural Science Foundation of Zhejiang Province (Y6110470),

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